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ABSTRACTS

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Combinatorial Results from Periodic Patterns of Signed Shifts

Given a map from a totally ordered set S to itself, one can define the pattern of length n associated with a point in S to be the permutation in the same relative order as the n-long initial part of the orbit. If we take this pattern of length n for a periodic point, then we get a periodic pattern. Using the periodic patterns of the signed shift maps on words, we can find interesting combinatorial results. For example, we get a bijection between periodic patterns and the cyclic permutations in some grid class (which can be characterized in terms of pattern avoidance). This bijection helps us enumerate the cyclic permutations in these classes.

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On the Computation of Foliations and Tracking Initial Conditions for Differential Equations

The Hartman-Grobman Theorem provides a local foliation for an ODE near a hyperbolic point; through all nearby points there are a pair of leaves that define a conjugacy to the linearized flow. In the classic case one leaf passing through the hyperbolic point is its unstable manifold, the other its stable manifold, both of which are invariant. In general the leaves are not invariant. They can, however, be characterized by the exponential growth/decay rates of the differences between solutions that start on them. If the gap in the spectrum of the linear part sufficiently dominates the Lipschitz condition of the nonlinear part in a large enough neighborhood, and the spectrum is positioned properly, the unstable manifold is an inertial manifold. Each solution is attracted at an exponential rate to a particular "tracking" solution on the inertial manifold. We present several algorithms for the accurate computation of the leaves in the foliation and as well as for the tracking initial condition for a given solution. The algorithms are demonstrated on the Kuramoto-Sivashinsky equation.

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Extremal H-colorings of graphs

An H-*coloring* of a finite, simple graph G is a map from the vertices of G to the vertices of a finite graph H (without multiple edges, but possibly with loops) that preserves edge adjacency. H-colorings generalize many important graph theoretic notions, such as proper q-colorings (via $H = K_q$) and independent sets (via H as an edge with a loop on one endvertex).

Consider the following extremal graph theory question: for a given H, which graph on n vertices with minimum degree δ has the largest number of H-colorings? In this poster I prove that the answer for many H is the complete bipartite graph $K_{\delta,n-\delta}$. I also provide graphs H where disjoint copies of $K_{\delta+1}$ or disjoint copies of $K_{\delta,\delta}$ yield the largest number of H-colorings.

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Adaptive design for non-stationary surfaces using changes in slope

Computer experiments are used to study physical processes that are too costly, difficult, or dangerous to experiment with in the physical world. Complex computer code that simulates these physical experiments often results in an extremely long running time. Hence, the design points must be chosen carefully and intelligently. An efficient design method that can investigate the response surface in a small number of samples is a must. With this is mind, sequential (adaptive) designs that allow users to focus their attentions on interesting areas of the response are a logical choice.

Historically, computer experiments included an assumption of stationarity, but new modeling methods have been shown to be effective at fitting non-stationary surfaces. We propose a sequential design method that is efficient at investigating non-stationary response surfaces. This method focuses on the search for areas with large changes in slope, with the idea that sudden changes in slope are an indication of non-stationary "breaks" in the response. While seeking out these boundary points, our method still achieves an effective fit of the entire response surface. The merits of this method are exhibited in several examples, including comparisons to existing sequential design methods.

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Theoretical model for metabolic blood flow regulation In a heterogeneous microvascular network

Blood flow in the microcirculation is regulated according to local metabolic demands of the tissue; however, the mechanism for this regulation is not entirely known. The purpose of this investigation is to analyze the eff ects of metabolic flow regulation by signals derived from red blood cells (RBCs), from the vessel wall, and from surrounding tissue, in response to changes in oxygen (O2) demand and delivery. A theoretical model is used to simulate blood flow, O2 transport, and flow regulation in microvascular networks with realistic heterogeneous structures. If O2 demand is increased or O2 delivery is decreased, the initial e ffects are increased regions of poor tissue oxygenation. If the metabolic signal is assumed to originate solely from a RBC-dependent mechanism, the model predicts that flow regulation will cause a reduction in blood flows and further worsening of tissue oxygenation. If the metabolic signal is assumed to originate instead from a wall- or tissue-dependent mechanism, flow regulation causes an increase in flow, resulting in improved tissue oxygenation. These findings suggest that a RBC-independent mechanism of metabolic blood flow regulation is required for an appropriate physiological response to changing O2 demand and delivery. NIH grant HL070657.

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Generalized Twistor Spaces for $K3$ Surfaces

Generalized complex geometry is a newly emerging field that unites two areas of geometry, symplectic and complex geometry, revealing surprising new aspects in both. Largely motivated by physics, generalized complex geometry provides a mathematical context for studying string theoretic concepts such as branes and supersymmetry. Both complex and symplectic structures can give rise to generalized complex structures, so it is natural to study $K3$ surfaces, which have complex and symplectic properties, in this context. In this poster, we use generalized complex geometry to construct a manifold fibred over $CP^1 \times CP^1$ that arises from a family of complex and symplectic structures on a $K3$ surface. Inspired by Penrose's twistor theory, we call this manifold a *generalized twistor space*. It is an interesting non-trivial example of a generalized complex manifold, and we discuss this as well as properties of the fibration over $CP^1 \times CP^1$ in this poster.

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Bilinear Littlewood-Paley Estimates and Caldern-Zygmund Theory

This work in harmonic analysis addresses the study of oscillatory behavior of functions in the context of bilinear operators. Bilinear operators are transformations that combine two waves into a new one. Some new almost orthogonality estimates are obtained, which provide understanding of interactions between waves oscillating at different frequencies. Using these estimates we are able to obtain new ways of quantifying properties of the resulting wave in terms of the initial waves. Estimates of this type are a bilinear version of Littlewood-Paley estimates and are used to justify useful frequency decompositions. Among other applications, our Littlewood-Paley estimates give a complete characterization of the continuity of certain operators called bilinear Caldern-Zygmund operators which are of great relevance in harmonic analysis.

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Covering Radius of Finite Geometry LDPC Codes

The covering radius of a code is a fundamental parameter that conveys how well a code covers the space that contains it. A low density parity check (LDPC) code is defined by a sparse parity check matrix, which allows such code families to be decoded efficiently using iterative algorithms. In general, LDPC codes lack the structure that allows calculations of exact values for the minimum distance and covering radius of a code. One notable family of structured LDPC codes, introduced by Kou, Lin, and Fossorier, is derived from the incidence relations of finite geometries. In this poster we present bounds on the covering radius of this family of LDPC codes and discuss various applications of these bounds.

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Programming with Mathematical Applications

We present innovative ideas from a newly created course in mathematical programming. This course was designed to enhance student understanding of mathematical concepts through algorithm construction, interactive graphics and modeling, teach basic programming and introduce the essentials of using technology to apply mathematical knowledge to real-world situations. Our particular emphasis with this poster is on the use of graphics to explore mathematical concepts such as Taylor series convergence and how it enriches the students overall learning experience. Joint work with Leslie Jones.

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Deforming cylindrical surfaces in hyperbolic manifolds by their harmonic mean curvature: Theoretical and numerical studies

We consider the harmonic mean curvature flow (HMCF) of an axially symmetric surface around a closed geodesic in Hyperbolic 3-manifolds. Assuming the initial surface is strictly convex and its HMC ¡ 1/2, we show that the evolving surface becomes cylindrical as it converges to the closed geodesic by obtaining optimal asymptotic estimates of both principal curvatures: $\lambda_1 \approx e^{-t}$ and $\lambda_2 \approx e^t$, thus $\lambda_1\lambda_2 \approx 1$. A numerical solution of this fully nonlinear parabolic PDE was also obtained by spectral methods. For axially symmetric surfaces, we verify that the numerical solution is in agreement with theoretical estimates and gives better curvature estimates not available from theoretical analysis. Numerical solutions of nonsymmetric surfaces will also be discussed.

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Multiscale Signatures of Beatles Songs in a Cover Song Task

While there are many Music Information Retrieval (MIR) classification tasks, this work focuses on the cover song task. Given a particular piece of music, the goal of the cover song task is to find all the different recordings of the given song by a variety of artists in a set of music. For this work, our dataset is a collection of songs by the Beatles. Our approach begins by building a novel multiscale signature for each song that captures repetitive structure at several scales, while also being of a manageable dimension. We then apply a metric that is appropriate for the cover song task to this representation space of song signatures, allowing for fine-tuned comparison between songs. This multiscale approach differs from those in the literature that largely consider single-scale, single-feature representations.

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Properties of Groupoid Dynamical Systems and Associated Crossed Product C^*-algebras

Given a locally compact Hausdorff groupoid G which acts on a C^*-algebra A, one can construct a new C^*-algebra $A \rtimes G$, called the crossed product of A by G. The groupoid crossed product directly generalizes the notion of a crossed product of a C^*-algebra by a locally compact group. Objects of the latter sort were studied extensively in the late 20th century, and we will discuss generalizations of certain known results to the groupoid case. In particular, we show that if A is a nuclear C^*-algebra and G is an amenable groupoid, then $A \rtimes G$ is itself nuclear. This is the analogue for groupoids of a theorem originally due to Philip Green. We will also investigate the implications that arise from assuming a slightly weaker condition, namely that A be an exact C^*-algebra.

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Heat Transmission Problems Across Fractal Mixtures:
Regularity Results and Numerical Approximation

We consider the numerical approximation of a two-dimensional second order parabolic transmission problem across a pre-fractal layer of Koch curve mixture type. The layer, a polygonal curve, divides a rectangular domain into two non-convex sub-domains. The approximation is carried out by a finite element method for the space variable and a finite difference scheme in time. There are two main difficulties in the numerical analysis of this type of problems, as well as in the simulations. On one hand, it is required to generate a suitable triangulation in order to achieve an optimal rate of convergence of the solutions. On the other hand, we must take into account and limit the intrinsic computational cost of numeric approximations. We obtained a priori error estimates. By exploiting the regularity results of the solutions and the self-similar feature of the pre-fractal interfaces, we developed a mesh refinement algorithm to create a family of nested conformal regular triangulations compliant with the so-called Grisvard's conditions, which allows to achieve an optimal rate of convergence both in space and in time.

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Home polytopes between regular polytopes
harmonic mean curvature: Theoretical and numerical studies

Hom-polytopes are the polytopes of affine maps between two convex polytopes. Their study is motivated by categorical analysis of polytopes a recent trend in this classical part of geometry. First steps towards a systematic theory were recently undertaken by Bogart-Contois-Gubeladze. Currently, our understanding is very limited even in the case of regular source and target polygons. We report on our joint ongoing project with J. Gubeladze on the hom-polytopes between higher dimensional regular polytopes. Counting their vertices is a blend of combinatorial, geometric, and arithmetic challenges.

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Spectrally Arbitrary Patterns and the Nilpotent Jacobian Method

A zero-nonzero pattern A is a matrix whose entries are from the set $\{*, 0\}$. We say that an $n \times n$ zero-nonzero pattern is a spectrally arbitrary pattern (SAP) over the field F if for every monic polynomial $p(x)$ with coefficients from F of degree n, there exists a matrix A over F with zero-nonzero pattern A such that the characteristic polynomial of A is $p(x)$. The Nilpotent-Jacobian Method is a powerful tool used to determine if a pattern is a SAP, when the field F is R. We will explore what (if any) information can be gleaned from this method when we look at a pattern over other fields, including finite fields, Q, $\overline{Q}$ (the algebraic closure of Q), and C.

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Efficacy of Human Papillomavirus Vaccine Evaluated through Mathematical Modeling

Approximately six million people are affected by Human Papillomavirus (HPV) every year. Currently, there are over half a million people affected with HPV worldwide. With over 100 different strains identified, HPV is the most common sexually transmitted diseases. I developed a stochastic model that describes the transmission of HPV infection in both men and women. I include screening and vaccination in the model and test different vaccination strategies using a stochastic differential equations model. Using published vaccine efficacy rates for the HPV vaccine, we find threshold vaccination levels for both men and women to reduce HPV infection in the population and speculate on cost-effectiveness of different vaccination strategies. These results emphasize the need for early detection and vaccination in BOTH genders to reduce total HPV morbidity in females. This work will be used inform and educate the public and public officials on the transmission of HPV and the need for vaccination in both genders.

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Polynomial Invariants in the Melvin-Morton-Rozansky Expansion of the Colored Jones Polynomial

Both the Alexander polynomial and the Jones polynomial are two well-known knot invariants. The Melvin-Morton conjecture, proved by Bar-Natan and Garoufalidis, provides a relationship between these two invariants. It states that the bottom 'line' in a certain expansion of the colored Jones polynomial generates the inverse of the Alexander polynomial. Rozansky later proved that the upper lines in this expansion generate rational functions with powers of the Alexander polynomial in the denominator. Furthermore, we get polynomial invariants of the knot in the numerator of these functions. We will describe these polynomials for some simple knots and a method for computing them.

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Speciality of linear systems based at fat points in P^n

We develop techniques for determining the dimension of linear systems of divisors based at a collection of general fat points in P^n by partitioning the monomial basis for $H^0(\mathcal{O}_{P^n}(d))$. The methods we develop can be viewed as extensions of those developed by Dumnicki. We apply these techniques to produce new lower bounds on multi-point Seshadri constants of P^2 and to provide a new proof of a known result confirming the perfect-power cases of Iarrobino's analogue to Nagata's Conjecture in higher dimension.

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Biological application for the oriented skein relation

The traditional skein relation for the Alexander polynomial involves an oriented knot, K_+, with a distinguished positive crossing; a knot K_-, obtained by changing the distinguished positive crossing of K_+ to a negative crossing; and a link K_0, the orientation preserving resolution of the distinguished crossing. We refer to (K_+, K_-, K_0) as the *oriented skein triple*.

A *tangle* is defined as a pair (B, t) of a 3-dimensional ball B and a collection of disjoint, simple, properly embedded arcs, denoted t. DeWitt Sumners and Claus Ernst developed the *tangle model* which uses the mathematics of tangles to model DNA-protein binding. The protein is seen as the 3-ball and the DNA bound by the protein as properly embedded curves in the 3-ball. *Topoisomerases* are proteins that break one segment of DNA allowing a DNA segment to pass through before resealing the break. Effectively, the action of these proteins can be modeled as $K_- \Leftrightarrow K_+$. *Recombinases* are proteins that cut two segments of DNA and recombine them in some manner. While recombinase local action varies, most are mathematically equivalent to a resolution, i.e. $K_\pm \Leftrightarrow K_0$. The oriented triple is now viewed as K_- = circular DNA substrate, K_+ = product of topoisomerase action, K_0 = product of recombinase action.

The theorem stated in this dissertation gives a relationship between two 2-bridge knots, K_+ and K_-, that differ by a crossing change and a link, K_0 created from the oriented resolution of that crossing. We apply this theorem to *difference topology* experiments using topoisomerase proteins to study SMC proteins.

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Equivalence of cyclic p^2 actions on handlebodies of genus g

We consider all orientation-preserving Z_{p^2}- actions, where p is prime, on 3-dimensional handlebodies V_g of genus $g \geq 2$. We study the graph of groups $(\Gamma, \mathbf{G})$, which determines a handlebody orbifold $V(\Gamma(\mathrm{v}), \mathbf{G}(\mathbf{v})) \simeq V_g/Z_{p^2}$. This algebraic characterization is used to enumerate the total number of Z_{p^2} group actions on such handlebodies, up to equivalence.

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An Improved Inequality Related to Vizing's Conjecture

Vizing conjectured in 1963 that $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ for any graphs G and H. A graph G is said to satisfy Vizing's conjecture if the conjectured inequality holds for G and any graph H. Vizing's conjecture has been proved for $\gamma(G) \leq 3$, and it is known to hold for other classes of graphs. Clark and Suen in 2000 showed that $\gamma(G\Box H) \geq \frac{1}{2}\gamma(G)\gamma(H)$ for any graphs G and H. This poster details a slight improvement of this inequality, proved by Suen and Tarr in 2010 by tightening their arguments.

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Weighted multi-parameter Hardy spaces associated with Zygmund dilations

Among the multi-parameter analysis, the Zygmund dilations are the simplest after pure product space dilations. We established the weighted multi-parmeter Hardy spaces $H^p_{\mathcal{Z}}(\omega)$ associated with Zygmund dilations for $0 < p < \infty$ and $\omega \in A_\infty(\mathcal{Z}) = \cup_{1\leq p<\infty} A_p$, the weighted class associated with the Zygmund dilation. And we proved the $(H^p_{\mathcal{Z}}(\omega), H^p_{\mathcal{Z}}(\omega))(0 < p < \infty)$ boundedness and the $(H^p_{\mathcal{Z}}(\omega), L^p_{\mathcal{Z}}(\omega))(0 < p \leq 1)$ boundedness of the Ricci-Stein multi-parameter singular integral operators for $w \in A_\infty(\mathcal{Z})$.

Moreover, we characterized the dual spaces of $H^p_{\mathcal{Z}}(\omega)$, that is, $(H^p_{\mathcal{Z}}(\omega))^* = CMO_{\mathcal{Z}}(\omega)$ for all $0 < p \leq 1$ and $\omega \in A_\infty(\mathcal{Z})$. Such Carleson measure spaces plays the same role as the John-Nirenberg BMO spaces in the duality $H^1(R^n) - BMO(R^n)$ in the non-weighted one-parameter setting when $p = 1$ and $\omega = 1$.

Our argument is based on the discrete Calderón reproducing formula and Littlewood-Paley-

Stein theory associated with the Zygmund dilations.